Combination of the Source Method with Absorbing Boundary Conditions in the Method of Lines

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Abstract—**The Method of Lines (MoL) is extended to improve the analysis of discontinuities in microstrip lines and antennas. Therefore the source method is developed by integrating absorbing boundary conditions in the reference plane. With the new approach a higher accuracy is reached by using fewer discretization points. This reduction of the numerical effort becomes necessary to develop effective software to analyze complex structures. The new procedure is demonstrated by calculating the end effect of an open-end. A comparison is then made between the results obtained by former and other methods. The improvement over the former procedure is made obvious.**

I. Introduction

THE method of lines (MoL) [1]—a special finite difference method—is one method for analyzing planar multilayered waveguide structures as used in microwave and millimeter wave integrated circuits. The MoL does not yield spurious modes and the relative convergence phenomenon is not observed.

With the source method described in [2], [3] it was possible to analyze discontinuities by using inhomogeneous boundary conditions in the plane of the feedline. With this approach the boundary was only transparent for the fundamental mode, as the other modes arising at the discontinuity were reflected. To obtain proper results, it was necessary to position the boundary containing the source a sufficient distance away from the discontinuity in order for the other modes to decay away. This procedure, however, caused a great increase in the amount of numerical effort required. A better approach for taking into account the total reflected and radiated field was the development of absorbing boundary conditions for the MoL [4]. Previously, these absorbing boundary conditions could only be applied to planar waveguides [5] or resonator structures [6]. In this paper a further development of the MoL by combining both methods, which has up until now only been applied to the calculation of input impedances of antennas [7], is demonstrated for the general analysis of discontinuities.

Different forms of discontinuities can be analyzed with this new procedure: discontinuities in strip and slot lines or coplanar guides, shielded structures and directly or indirectly fed patch antennas. In order to verify the procedure the analysis of a microstrip open-end is presented in this paper. An open-end is a simple structure, but widely used in a number of integrated circuits such as resonators or filters. It has been carefully examined with different methods so that the results are easily compared with those from other methods of analysis.

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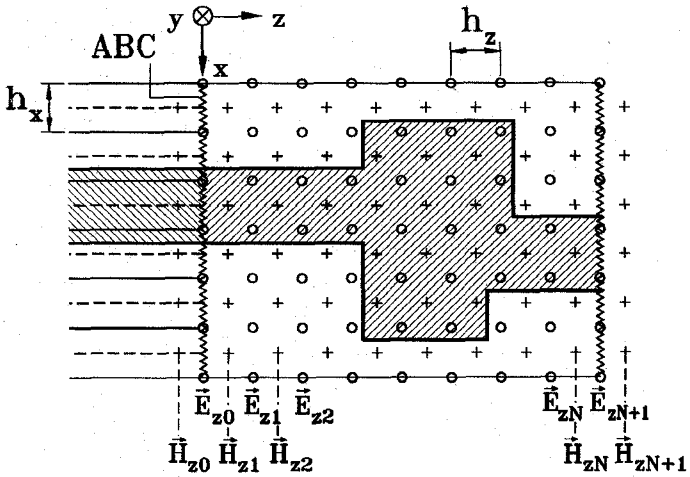

Fig. 1. Discretization.

II. Theory

The analysis of a three dimensional structure (see Fig. 1) with negligible metallization thickness is made according to [1] with a two-dimensional discretization. But in contrast to the former method a different treatment of the z direction is performed to consider the reflected waves arising at a discontinuity.

The potentials near the plane of the feedline are separated into an incoming and a reflected part. The difference operators for both parts are constructed differently. Absorbing boundary conditions (ABC's) are applied to the outgoing waves which result from the reflection at the discontinuity, whereas the incoming field distribution is calculated from the fundamental mode of the one-dimensional feedline problem

$$\vec{E}_{zn} = \vec{E}^{+}_{zn} + \vec{E}^{-}_{zn}, \quad n = 0, 1, 2, 3, 4 \tag{1}$$

$$\vec{E}^{+}_{zn} = \vec{E}_{zL} \cdot e^{-j\sqrt{\varepsilon_{re}}\bar{h}n} \tag{2}$$

with $\vec{E}_{zL}$: solution of the line problem.

For the outgoing part we use the modified difference operator $\boldsymbol{D}_{za}$ containing the coefficients for the ABC's

$$\overline{h}_z \left.\frac{\partial e_z}{\partial \overline{z}}\right|_{z_i} \longrightarrow \underbrace{\begin{bmatrix} a_1 & -b_1 & -c_1 & & \\ -1 & 1 & & & \\ & \ddots & \ddots & & \\ & & & -1 & 1 \\ & c_N & b_N & -a_N & \end{bmatrix}}_{\boldsymbol{D}_{za}} \begin{bmatrix} E^t_{z1} \\ \vdots \\ E^t_{zN} \end{bmatrix} \quad (3)$$

These coefficients $a_{1(N)}$, $b_{1(N)}$ and $c_{1(N)}$ are described in [9] in detail. But it must be mentioned that a typographical error occurred in the description. In the formulas for these coefficients, the sign of the term $\overline{n}_d^{-1}$ has to be changed throughout. For radiation problems the formulas of the improved special absorbing boundary conditions (SABC's) can also be used for normal absorption by substituting $\overline{n}_d$ with $\overline{n}_a = j\overline{h}n_{t,b}$.

Here and in what follows all coordinates and the discretization distances h_x and h_z are normalized according to $\overline{x} = k_o x$ etc., where k_o is the free space wave number.

The resulting difference expression for the z direction reads as follows:

$$\begin{aligned} \overline{h}_z \frac{\partial e_z}{\partial \overline{z}} &\rightarrow [\vec{\boldsymbol{E}}^-_{z1}, \vec{\boldsymbol{E}}^-_{z2}, \vec{\boldsymbol{E}}^-_{z3}, \vec{\boldsymbol{E}}^-_{z4}, \vec{\boldsymbol{E}}_{z5}, \cdots, \vec{\boldsymbol{E}}_{zN}]\boldsymbol{D}^t_{za} \\ &\quad + [\vec{\boldsymbol{E}}^+_{z1}, \vec{\boldsymbol{E}}^+_{z2}, \vec{\boldsymbol{E}}^+_{z3}, \vec{\boldsymbol{E}}^+_{z4}, \vec{0}, \cdots, \vec{0}]\boldsymbol{D}^{t+}_z \\ &= \boldsymbol{E}^-_z \boldsymbol{D}^t_{za} + \boldsymbol{E}^+_z \boldsymbol{D}^{t+}_z. \end{aligned} \quad (4)$$

$\boldsymbol{E}^-_z$ and $\boldsymbol{E}^+_z$ are matrices. The operator $\boldsymbol{D}^{t+}_z$ is constructed with

$$a^+ = 1 - e^{j\sqrt{\varepsilon_{re}}\overline{h}_z} \quad (5)$$

$$\boldsymbol{D}^{t+}_z = \begin{bmatrix} a^+ & -1 & & \\ & 1 & \ddots & \\ & & \ddots & \ddots \\ & & & 1 & -1 \end{bmatrix}. \quad (6)$$

The second derivative is obtained through multiplication with the difference operator $\boldsymbol{D}_z$

$$\overline{h}^2_z \frac{\partial^2 e_z}{\partial \overline{z}^2} \rightarrow -\boldsymbol{E}^-_z \boldsymbol{D}^t_{za} \boldsymbol{D}_z - \boldsymbol{E}^+_z \boldsymbol{D}^{t+}_z \boldsymbol{D}_z. \quad (7)$$

It should be noted that the splitting of the field is only necessary in the first four columns of the matrices, which is correlated to the number of coefficients of the ABC's. In the other columns the matrix $\boldsymbol{E}^-_z$ contains the whole field, and the matrix $\boldsymbol{E}^+_z$ is a null matrix.

With the new difference operator we obtain the wave equation for the whole field. For the incoming components the wave equation of the line problem is valid, so that the wave equation for the reduced part reads as

$$\begin{aligned} \frac{d^2\boldsymbol{E}^-_z}{d\overline{y}^2} + \varepsilon_r \boldsymbol{E}^-_z - \boldsymbol{D}^t_{\overline{x}} \boldsymbol{D}_{\overline{x}a} \boldsymbol{E}^-_z - \boldsymbol{E}^-_z \boldsymbol{D}^t_{\overline{z}a} \boldsymbol{D}_{\overline{z}} \\ = \boldsymbol{E}^+_z \boldsymbol{D}^{t+}_{\overline{z}} \boldsymbol{D}_{\overline{z}} - \varepsilon_{re} \boldsymbol{E}^+_{\overline{z}}. \end{aligned} \quad (8)$$

The bars on x and z in the difference operators $\boldsymbol{D}_{\overline{x}}$ etc. indicate that they are normalized by $\overline{h}_x$ etc. To be able to solve the inhomogeneous differential equations for $\overline{y}$, a transformation according to $\boldsymbol{E}^-_z = \boldsymbol{T}_{xe} \overline{\boldsymbol{E}}^-_z \boldsymbol{T}^{-1}_{ze}$ is performed with

$$\boldsymbol{T}^{-1}_{ze} \boldsymbol{D}^t_{\overline{z}} \boldsymbol{D}_{\overline{z}a} \boldsymbol{T}_{ze} = \overline{\boldsymbol{\lambda}}^2_{ze}. \quad (9)$$

Summarizing the terms on the right side leads to

$$\frac{d^2\overline{\boldsymbol{\psi}}^-}{d\overline{y}^2} + \varepsilon_r \overline{\boldsymbol{\psi}}^- - \overline{\boldsymbol{\lambda}}^2_{\overline{x}e,h} \overline{\boldsymbol{\psi}}^- - \overline{\boldsymbol{\psi}}^- \overline{\boldsymbol{\lambda}}^2_{\overline{z}e,h} = \overline{\boldsymbol{\psi}}_{LG} \boldsymbol{X}_{e,h} \quad (10)$$

with $\boldsymbol{\psi} = \boldsymbol{E}_z$ or $\boldsymbol{H}_z$. (The Neumann condition is treated similarly, hence it is not shown here.)

$\overline{\boldsymbol{\psi}}_{LG}$ contains the solution of the line problem in each column, whereas the diagonal matrix $\boldsymbol{X}_{e,h}$ is obtained by combining the other factors.

In this form the inhomogeneous wave equation can easily be solved with the particular solution

$$\overline{\boldsymbol{\psi}}_P = \overline{\boldsymbol{\psi}}_{LG} \boldsymbol{X}_{e,h} (\overline{\boldsymbol{\lambda}}^2_z - \varepsilon_{re} \boldsymbol{I}_z)^{-1}. \quad (11)$$

The tangential components in the metallization plane are calculated from

$$\left(\frac{\partial^2}{\partial \overline{z}^2} + \varepsilon_r\right) \begin{bmatrix} e_x \\ h_x \end{bmatrix} = \begin{bmatrix} \frac{\partial^2}{\partial \overline{x} \partial \overline{z}} & -j\frac{\partial}{\partial \overline{y}} \\ -j\,\varepsilon_r \frac{\partial}{\partial \overline{x}} & \frac{\partial^2}{\partial \overline{y} \partial \overline{z}} \end{bmatrix} \begin{bmatrix} e_z \\ h_z \end{bmatrix}. \quad (12)$$

Through the matching at the interface, a deterministic equation for the current distribution on the metallization is derived. The first columns in this distribution contain only the reflected parts. The amplitude of the fundamental mode is obtained by filtering out from the total field using the relation of orthogonality for the different modes of the current distribution, which is valid for a lossless waveguide.

With this the input impedance is calculated directly from the reflection coefficient using the incoming current known from the line problem.

$$r = -\frac{\vec{J}^-_z \vec{J}^+_z}{|\vec{J}^+_z|^2}. \quad (13)$$

The transmitted fundamental mode and the transmission parameter S_{21} can be calculated from the scattered field at the output port. To obtain the scattering parameter for the other direction, a source at the output port is assumed. Because of the linearity of the field equation and the materials the superposition rule holds. Therefore this case can be considered separately.

III. RESULTS

To verify the described procedure, the end effect of an open microstrip end was calculated. For the given structures the results are compared to the results of [8] who used the MoL with the conventional source method, and [10]

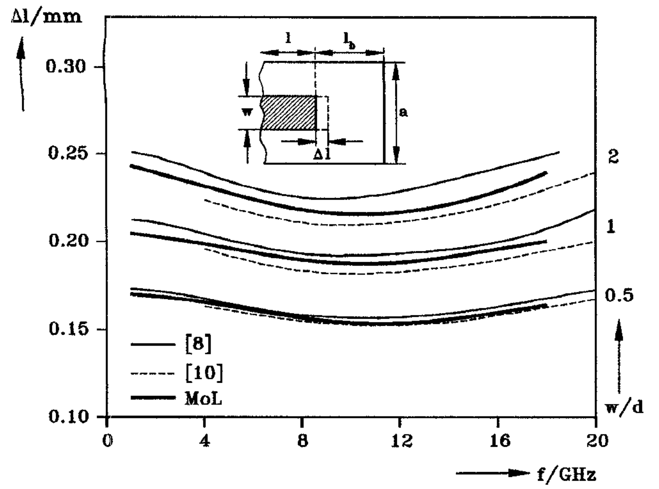

Fig. 2. End effect dependent on frequency. $\varepsilon_r = 9.7$, $d = 0.635$ mm, $a = w + 10d$, $l_b = 5$ mm.

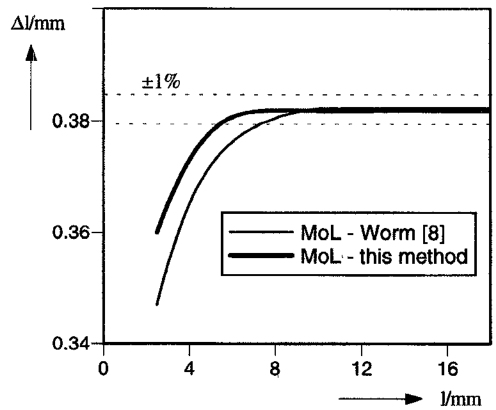

Fig. 3. Investigation of convergence. $\varepsilon_r = 2.3$, $d = 0.79$ mm, $a = w+ 10,d$ $l_b = 5$ mm, $w/d = 2$, $f = 4$ GHz.

with the Galerkin method (see Fig. 2). In contrast to [8] a nonequidistant discretization [11] was used for all calculations.

As can be seen, the new results have only a small deviation with respect to the other ones. The maximal deviation is about 6%. In comparison to the source method of [8], the results are gained with a much smaller numerical effort because the reference plane was situated closer to the end. This allows the region to be discretized to decrease very much in size. To have a good convergence of the results, [8] had to choose the minimal length of the microstrip end to $l = 7.5$ mm. With the new procedure an investigation of convergence was made as well (see Fig. 3).

Here the results converge by using a smaller length and—corresponding—a smaller number of points.

IV. Conclusion

This paper demonstrates how the source method is extended by using absorbing boundary conditions.

The obtained results are in good agreement with the results of [10]. They are better than the results of Worm, who needed larger storage capacities and more computer time, because he had to position the reference plane in a greater distance away for the higher order modes to decay. With the new method the reference plane is transparent to these modes, so that the distance from the discontinuity can be smaller. Saving of time and memory is necessary to increase the efficiency of the method in order to analyze not only single elements but also more complex and coupled structures, which are of greater technical interest. The procedure shown can also be extended to any other kinds of shielded structures such as coplanar or slot lines, as well as to fed antennas. A detailed publication about the application to antennas is in preparation.

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